The Mathematics and Applications Behind Image Warping and Morphing

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Abstract

The purpose of this research is to reveal the mathematics and applications of the computer animation techniques of warping and morphing. A warp is a twist or distortion in the form of an object in an image while a morph is the smooth and gradual transformation of an object in one image into the object in another image. Linear algebra makes these computer animation techniques possible; the first phase of this research delves into how those mathematical processes translate into image warps and morphs. The second part of this study requires the analysis and application of image warping and morphing techniques in an array of fields. The team utilized the computer software, *Abrosoft Fantamorph*, in order to create a series of warps and morphs. The final phase of this research was to identify what uses NASA can have for these computer animation techniques and what further research can be done to expand our knowledge of warps and morphs. By identifying the mechanics of warps and morph, we can discover how they can assist scientists and organization, such as NASA, to create depictions of objects, ideas, places, and events. Ultimately, studying morphing and warping techniques allows us to find better ways to represent visual data - whether it is images of the ozone hole or maps of the ever-changing weather in a region. The limitations that were found during the study can be used to conduct further research about warps and morphs - such as distorting images using quadratics or varying the rate at which each part of a transformation happens.

Introduction

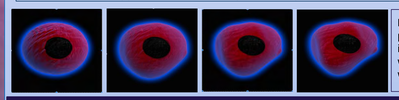
Have you ever wondered about how the 3D representations of the moon are created? Or how images from different sources - especially satellites - are combined to form uniform and cohesive images? Or even how the gradual process of a person aging can be depicted from just two images? Such are the questions that can be answered as a result of an in depth analysis of warping and morphing techniques and the mathematics and applications behind them.

In definition, warps and morphs are computer animation techniques that depict the distortion, manipulation, or transformation of images. To be more specific, a warp is a twist in the form or shape of an image. An example of this is distorting the image of a chicken by elongating its legs. Warping two complementary objects by forcing them to blend and assume the form of one another results in a morph - which is the smooth and gradual transformation of the image of one object into that of another. An example of a morph is transforming the image of a chicken into the image of a dinosaur. One of the biggest differences between a warp and a morph is that while a warp only requires one picture, a morph requires at least two pictures - to create longer and more complicated morphs, more images are necessary. The mathematics behind these computer animation techniques will be discussed later in the research, however, it is important to note that the process of a warp or morph is purely dependent on linear algebra.

In essence, a morph is two warps happening at the same time, in a synchronized fashion. The beginning image is warped with respect to the end image and vice versa. An example showing the mechanics of a morph is shown below:

Both images are manipulated so that each is warped in the direction of the other. As the objects in the images assume a different form, they are overlapped and their color densities are averaged in accordance to the time. The example shown above follows the morphing process of a bird as it transforms into a prairie dog. The red shaded area in the second image indicates the change in its shape that the bird had to undergo in order to morph into the prairie dog. A similar process is happening in the opposite direction; the prairie dog's form is altered so that it assumes the shape of the bird. While this shape transformation is happening, the color densities of corresponding pixel are averaged in a specific way. For example, assuming the morph happens within the time frame of t=0 and t=1, at t=0.25, the color density of the first picture would account for three-fourths of the colors in that specific frame, while the color density of the second picture would only account for one-fourth of the color in the frame. The color densities of the rest of the frames in the morph would be calculated in the same way.

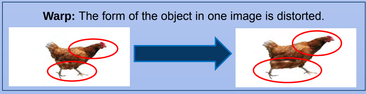
The processes of warps and morphs can be instigated by using specially-designed computer software. The method that is used to create the specific warps and morphs described above entails a linear process in which the rates at which the objects in the images change are constant. The mathematics and applications behind warps and morphs reinstate how incredibly useful they can be.



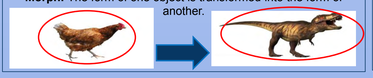
Left: Though the most visible applications of warps and morphs are for the production of special effects in film and television, there is a range of other important applications. One of the first successful morphs the team generated was one that depicted the transformation of a normal cell into a cancer cell. In this way, morphing and warping techniques can be utilized in the medical field.

Methods

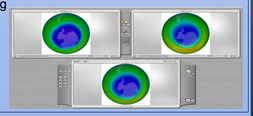
Warp: The form of the object in one image is distorted.



Morph: The form of one object is transformed into the form of another.



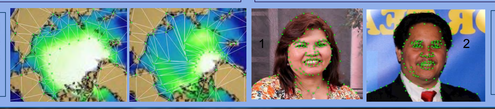
The team used a software called *Abrosoft Fantamorph* to experiment and create a series of image warps and morphs. The software utilizes mathematical means - such as triangulation - in order to apply warp/morph techniques such as generating smooth transitions between different images and creating distortions in the forms of objects in selected images. The team used the *Delux* version of the software in order to view the process of triangulation as the images were warped or morphed.



Warps Procedure

1. Input points at key features
2. Create a triangular mesh over all the points
   – Same mesh in both images
   – Points = Vertices
   – Triangle-to-triangle correspondences
3. Through an affine transformation, each point is separately mapped from the first picture (the source) to the second picture (the destination).

A warp is created when a point from the original image is moved to a different location.

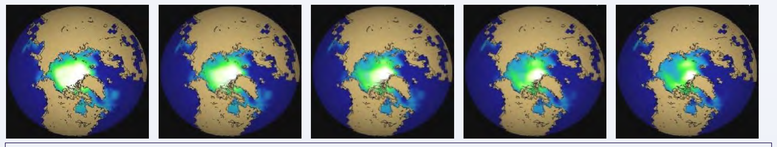


Morphs Procedure

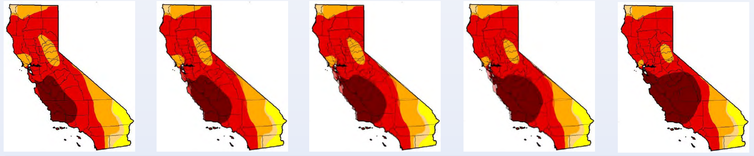
1. Input points on defining features in image 1.
2. Create a mesh of triangles (triangulation) from the points
   – Both images 1 and 2 are covered by a congruent mesh of vertices and triangles.
3. Move the points in the second image to desirable/corresponding locations.
   –Avoid placing points that cause overlapping triangles

A morph is created when points in image 2 are placed to the corresponding places in new locations.

Applications of Warps/Morphs

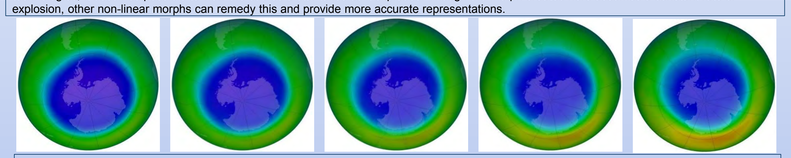


ABOVE: Through the use of the image morphing software, it is possible to create a comparison depicting the gradual changes in sea ice thickness. The above comparison shows the dramatic decrease of sea ice thickness in the Arctic region from the 1950s to its projected thickness in 2050 (this estimate is based on reasonable assumptions of climate change). In fact, more recent analyses suggest that this projection might actually be optimistic. The white area shows the thicker sea ice and the green depicts thinner ice.

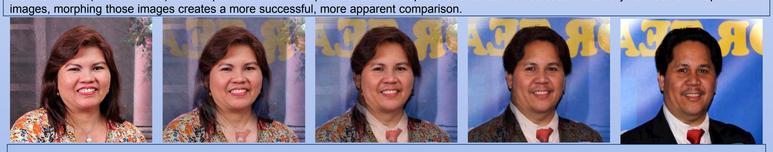


ABOVE: Image morphing is also useful in mapping the extent of natural disasters, such as droughts. In this series of frames (extracted from a morph), it is possible to see the rate of change of drought intensity in California over a one week span. The dark red in the map depicts the areas with the highest drought intensity. By creating a morph of this change, it becomes easier to analyze the extent and intensity of a drought - or any other natural disaster for that matter - on a given piece of land.

ABOVE: On February 23 1987, a star exploded in the Large Magellanic Cloud - a nearby galaxy. NASA was able to obtain images of the supernova before and after its explosion. The frames from a morph that depicts the explosion of the star are shown above. Through image morphing technologies, it becomes possible to view the course a star takes as it explodes. Though the morph created here assumes a constant rate of explosion, other non-linear morphs can remedy this and provide more accurate representations.



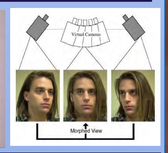
ABOVE: Some changes in a vector are hard to view with the naked eye. Morphing images that depict slight changes can create better and easier-to-view depictions of those vectors. In the frames shown above, morphing software has been used to illustrate the slight decrease in the size of the ozone hole (shown in blue) from September 2006 to September 2012. Despite the fact that this decrease is not very noticeable in separate images, morphing those images creates a more successful, more apparent comparison.



ABOVE: Morphing technologies are also popularly utilized to create morphs between different humans or to depict the gradual aging process of one human. Morphing family members - or any humans for that matter - can be used to compare similarities or differences in facial structure and body form. The aging process of one person can also be visually depicted from only two images.

Another useful application of morphing software is the possibility to create 3D illusions that are reminiscent of utilizing a virtual camera. After taking images from various angles of an object, one can separately morph consecutive images. The next step is to collate these individual morphs. As a result, when observing the result - the "master" morph - one is given the illusion of viewing a virtual camera. The 3-dimensional illusion is thus created by morphing 2-dimensional images. This significant implication can be used in an array of fields; we can create virtual camera and 3D depictions of landforms, planets, humans, animals, objects, locations, inventions, machines, and more.

3D Illusions



Relevance to NASA

The National Aeronautics and Space Administration can use different warping and morphing techniques for a plethora of reasons: to depict environmental changes on Earth, to create better representations of extraterrestrial objects by transforming 2D satellite images into 3D virtual camera views, to combine images captured by different satellites to form more uniform and cohesive pictures of objects, to depict the gradual process that an object undergoes within the span of time the two images are taken, to emphasize any slight or easily overlooked changes an object experiences, to collate images that depict a single object with respect to time, and more. Distorting images can also be used to create projections. For example, NASA can warp an image of a map of Florida by manipulating the coastline in accordance to how scientists predict the coast of Florida will look like in a few decades due to inevitable sea level rise. Morphing techniques can depict how this process will look like over the specific span of time corresponding to this projection. Furthermore, NASA can create 3D virtual camera views of planets such as Mars and Pluto by morphing 2D photographs of these planets taken by satellites at different angles. This morphing technique will allow NASA to obtain more cohesive depictions of these planets and other extraterrestrial objects. The beauty of warping and morphing techniques is that they have an impressive array of applications and uses for the purpose of exploring change and depicting objects.

Mathematics of Warps/Morphs: Linear Algebra

Computer graphics software allows for the manipulation of images in various ways - rotating, dilating, shearing, scaling, cropping, slanting, etc. Using linear algebra, computer software can also be developed to create warps and morphs. A summary of the math which is used for these techniques is outlined below.

Warps and morphs are examples of affine transformations. An affine transformation is any type of transformation that preserves the ratio of distances between the points and the collinearity of each of the points. In essence, any triangle is capable of being transformed into another triangle by the means of an affine transformation. As a result, all triangle are affine.

To create a simple warp, we must first construct a triangular region on a plane by choosing three noncollinear points, where:

$$V = C_1V_1 + C_2V_2 + C_3V_3$$

This triangle from the first image is mapped on to a new plane, where:

$$W = C_1W_1 + C_2W_2 + C_3W_3 \quad \text{and:} \quad C_1 + C_2 + C_3 = 1$$

C represents a unique constant. After the entire vector (the first image) is triangulated, a warp - or a morph - can only be successfully generated if these condition are satisfied:

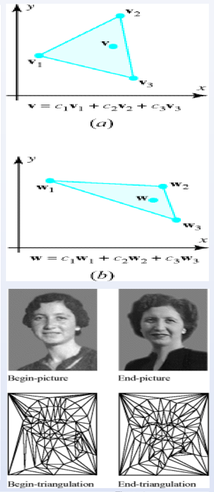
1. The line segments form the sides of a set of triangles.
2. The line segments do not intersect.
3. Each vertex point is the vertex of at least one triangle.
4. The union of the triangles is the rectangle.
5. The set of triangles is maximal (i.e., no more vertices can be connected).

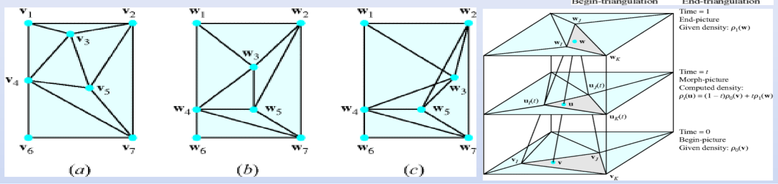
Because a morph is two warps that are combined, the math behind warps is a major part of the math behind morphs. However, in a morph, the densities of the colors in the two images are also being averaged. The color density of any two corresponding points on the images that are being morphed is averaged by the following equation:

$$p_t(u) = (1 - t)\, p_0(v) + t p_1(w)$$

Where: $p_1(w)$ = End Picture's Density *and* $p_0(v)$ = Begin Picture's Density

The main color sequence that is averaged is comprised of red, blue, and green (RBG). The entire morphing/warping process is illustrated to take place between t=0 and t=1.





Discussion and Future Research

After analyzing the various warps and morphs that the team generated using the *Abrosoft FantaMorph* software, it is reasonable to conclude that computer warping and morphing techniques are indeed incredibly useful in an array of fields. Whether it is the mapping of global climate change or simply the 3D representation of a planet, these techniques play an essential role in creating depictions of important concepts and bettering our understanding of change - from the past to the present to the future. At the conclusion of this study, it is clear that the mathematics behind warps and morphs have a direct connection to how these techniques are translated to images and that both of these computer imaging techniques can be applied to a plethora of fields and disciplines.

Though these techniques have quite a number of useful applications, unfortunately, they are also comprised of a small number of limitations. For example, because some morphs can only be instigated in a linear fashion and the change between each frame in the morph happens at a constant rate. As a result, the gradual process that an object undergoes as it transitions from the first image to the second image may not be accurately represented. Numerous natural processed to do not happen in a linear, constant pattern, so it would be inaccurate to presume that morphs created of the images depicting hose processes are error-free. To remedy this, software that can morph objects in trigonometric or quadratic fashions might be useful. Furthermore, an additional major shortcoming the team members experienced while generating morphs was overlapping triangles during the relocation of points. If an object undergoes dramatic changes in which it loses or gain new parts, it is inevitable that the triangles will overlap at some point of the morphing process. As a result, it is simply impossible for some objects to be morphed between images. One of the only methods to resolve this problem is to divide the complete image of the object into smaller images and consequently, morph each part of the image separately. Finally, one would collate all the separate morphs together to run them at the same time. Though there are more possible limitations of using image warping and morphing techniques, these two are of paramount importance.

In order to take this research further, teams can analyze the functions of warps and morphs that do not work in linear fashions. While this study analyzes the mathematics and applications of the most simple type of morph - linear morphs (in which image frames morph at a constant rate) - it is also crucial to study the mathematics and applications behind other varieties of warping/morphing techniques in order to gain better understanding and and to create depictions of objects that do not transform in linear manners. Moreover, further research can also concentrate on the roles of warps and morphs in creating projections.This research would play a significant role in assisting scientists and organizations, such as NASA, to predict future occurrences.

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