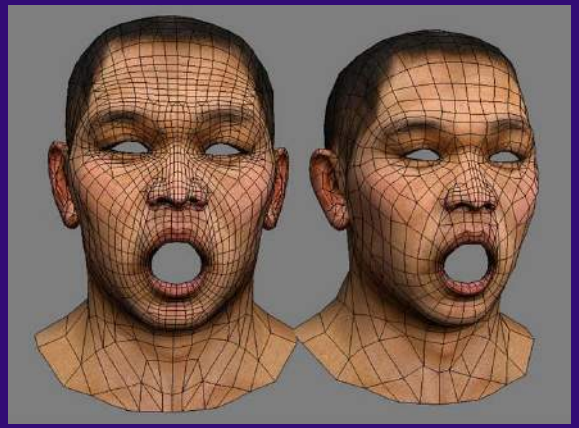
The Mathematics and Applications Behind Image Warping and Morphing

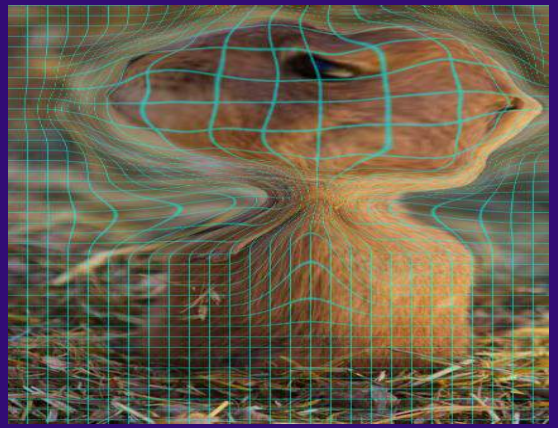
NASA NYCRI: Hostos Team
Prof. Tanvir Prince (Mentor)
IIdefonso Salva (High School Teacher)
Maria Malik (High School Student)
Ariel Mazor (Undergraduate Student)

Black or White Face Morphing by Michael Jackson

https://www.youtube.com/watch?t=32&v=4jd6dNrJRh4

What are image warps and morphs?





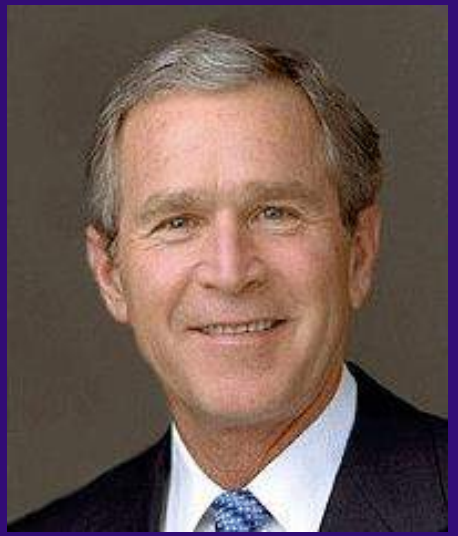
Computer animation techniques
Manipulation, distortion, or transformation of images

Image Warp:

A distortion or twist in the form or shape of an image.

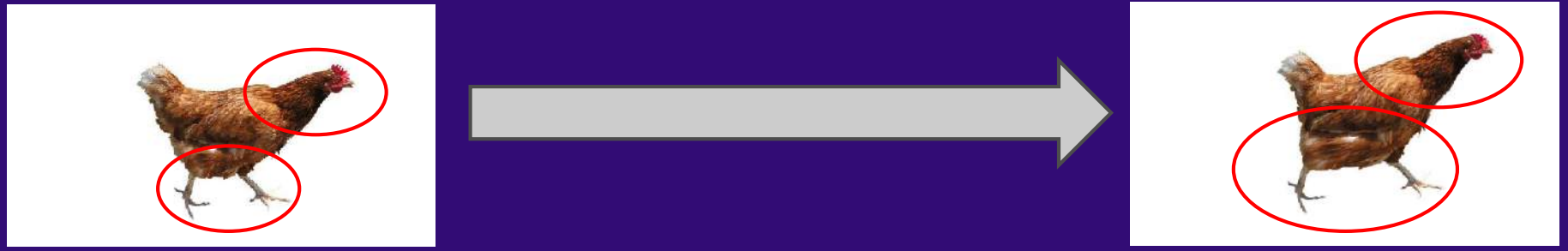
Image Morph:

The smooth and gradual transformation of the image of one object into that of another.

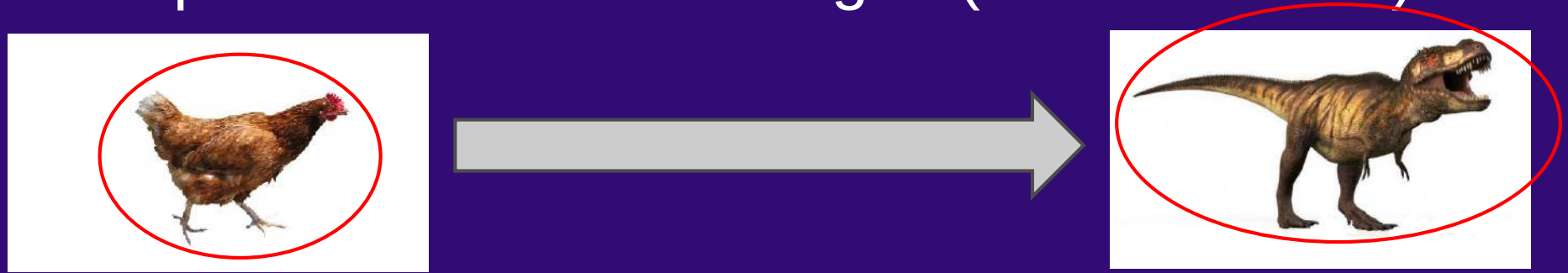


ONE MAJOR DIFFERENCE BETWEEN WARPS AND MORPHS:

- Warps: ONE image (Same image is changed)



- Morph: TWO different images (Start and End)



Mathematics Behind Warps

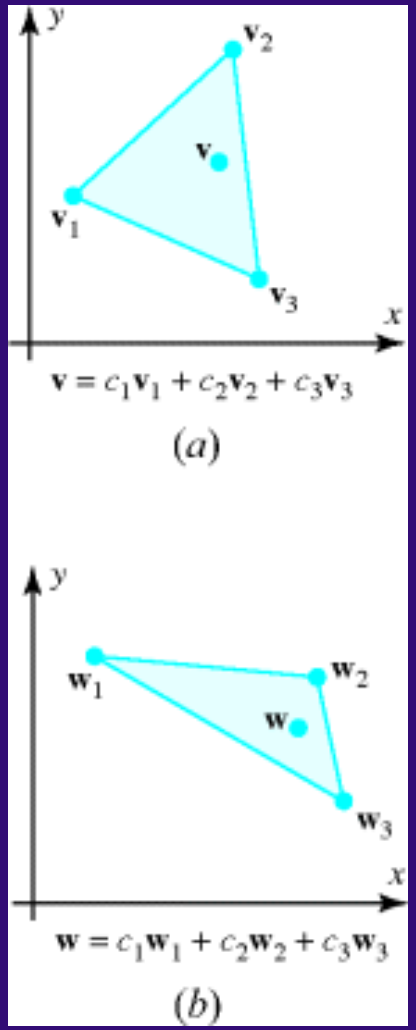
To create a simple warp, we must first construct a triangular region on a plane by choosing three noncollinear points, where:

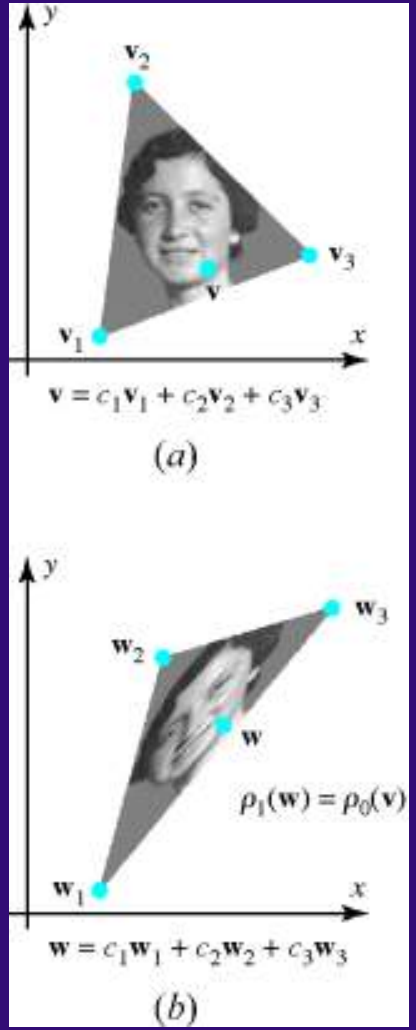
$$V = C_1V_1 + C_2V_2 + C_3V_3$$

This triangle from the first image is mapped on to a new plane, where:

$$W = C_1W_1 + C_2W_2 + C_3W_3$$

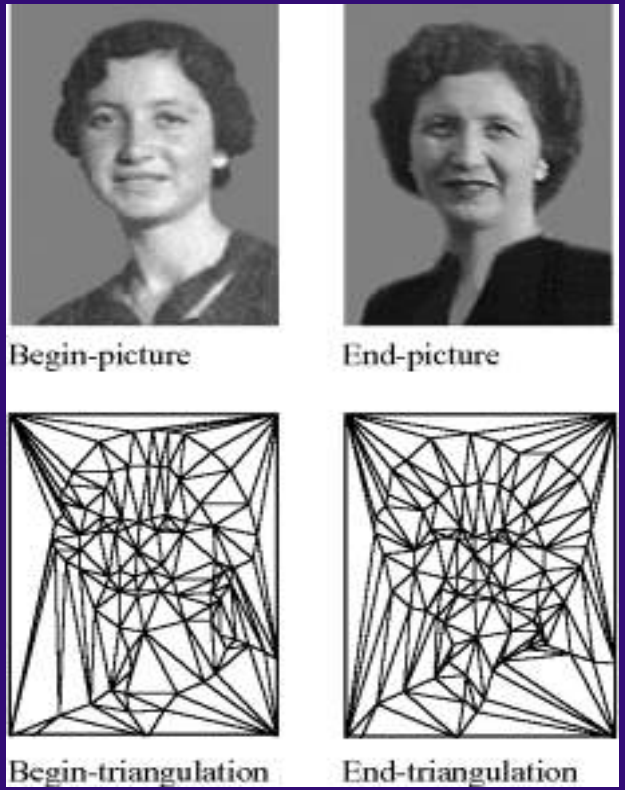
and: $C_1 + C_2 + C_3 = 1$

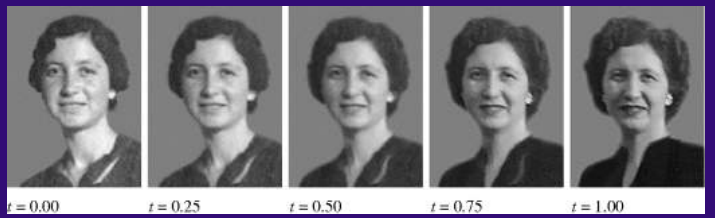




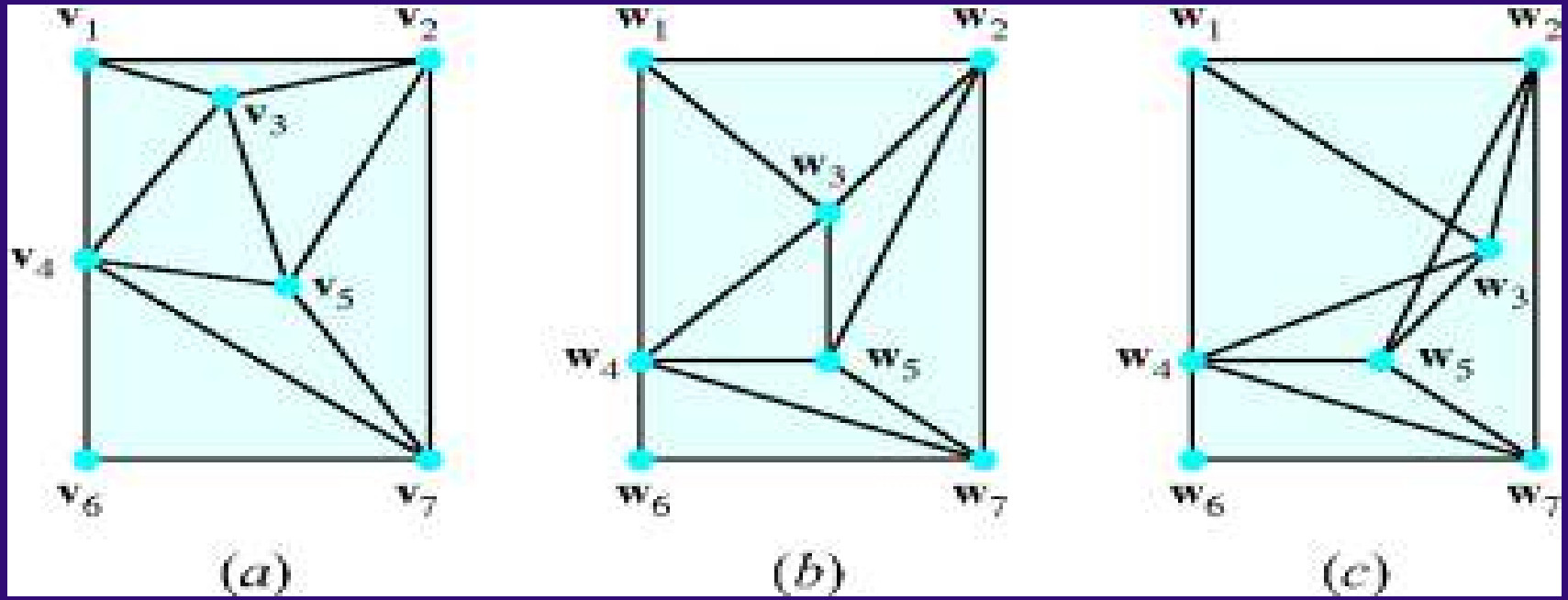
After the entire vector (the first image) is triangulated, a warp - or a morph - can only be successfully generated if these condition are satisfied:

1. The line segments form the sides of a set of triangles.
2. The line segments do not intersect.
3. Each vertex point is the vertex of at least one triangle.
4. The union of the triangles is the rectangle.
5. The set of triangles is maximal (i.e., no more vertices can be connected).





Triangulation

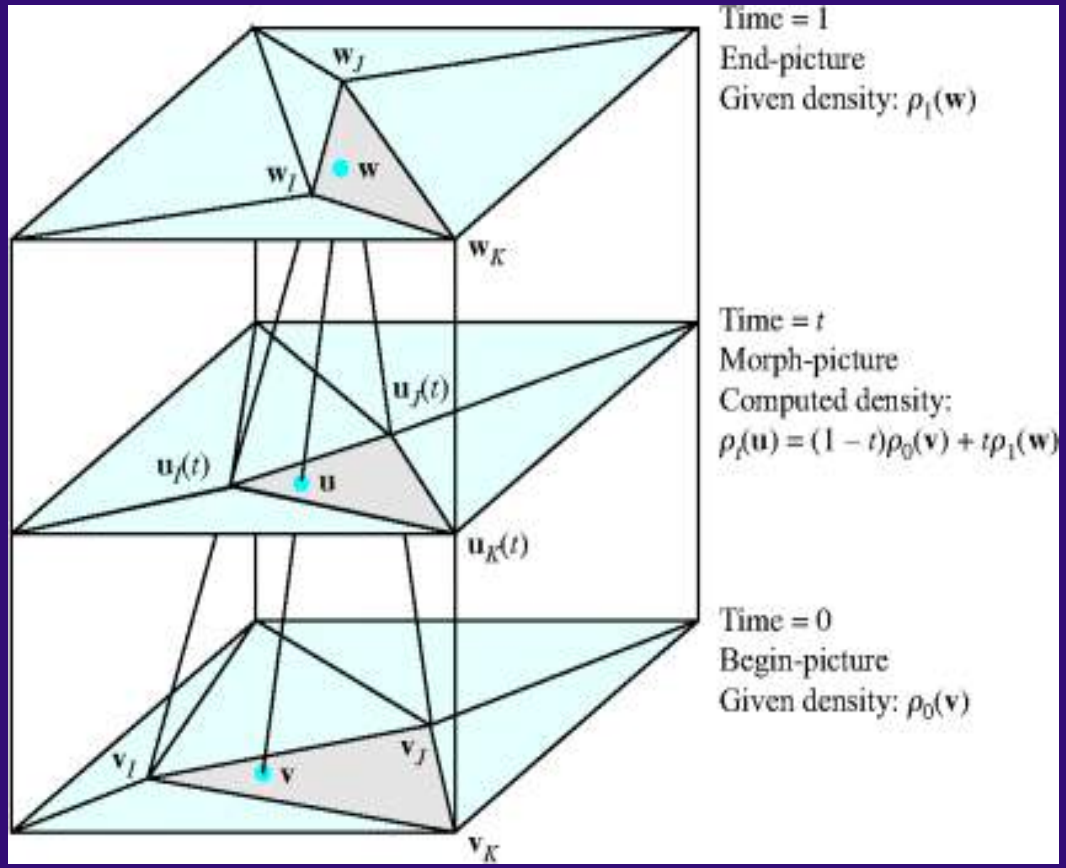


(a) First Phase: Chosen triangles

(b) Second Phase: Triangles are distorted

(c) The triangles must not overlap. This will cause lapses in warps/morphs.

Averaging Picture Density in Morphs



The color density of any two corresponding points on the images that are being morphed is averaged by the following equation:

$$p_t(u) = (1 - t)\, p_0(v) \;+\; t p_1(w)$$

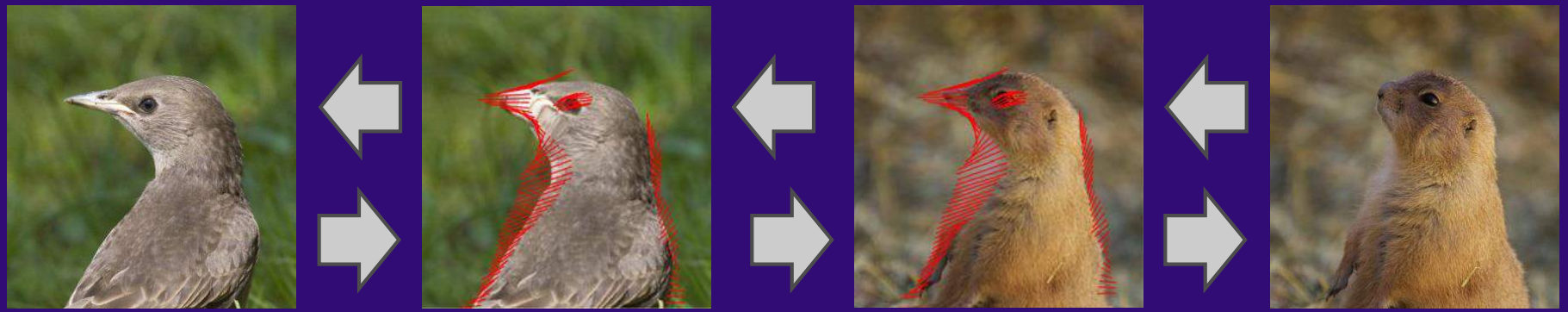
Where:

$p_1(w)$ = End Picture's Density

$p_0(v)$ = Begin Picture's Density

The main color sequence that is averaged is comprised of red, blue, and green (RBG).

In essence, a morph is TWO warps happening at the same time.

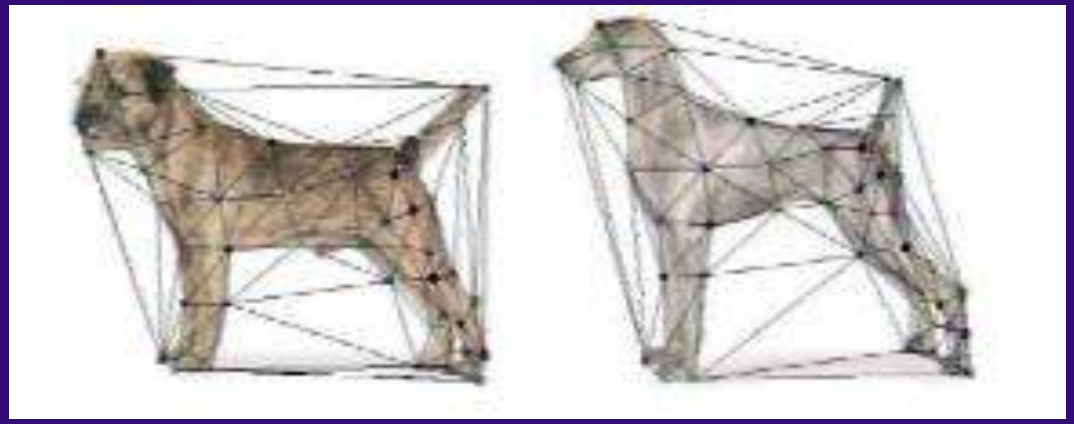


The beginning image is warped with respect to the end image and vice versa (image 1 is warped so that it transitions into the shape of image 2).

The warping frames are overlapped and their color densities are averaged in accordance to the time.

T=0.00 T=0.25 T=0.50 T=0.75 T=1.00

Procedure to Create a Warp



1. Input points at key features
2. Create a triangular mesh over all the points
   - Same mesh in both images
   - Points = Vertices
   - Triangle-to-triangle correspondences
3. Through an affine transformation, each point is separately mapped from the first picture (the source) to the second picture (the destination).

The Result……

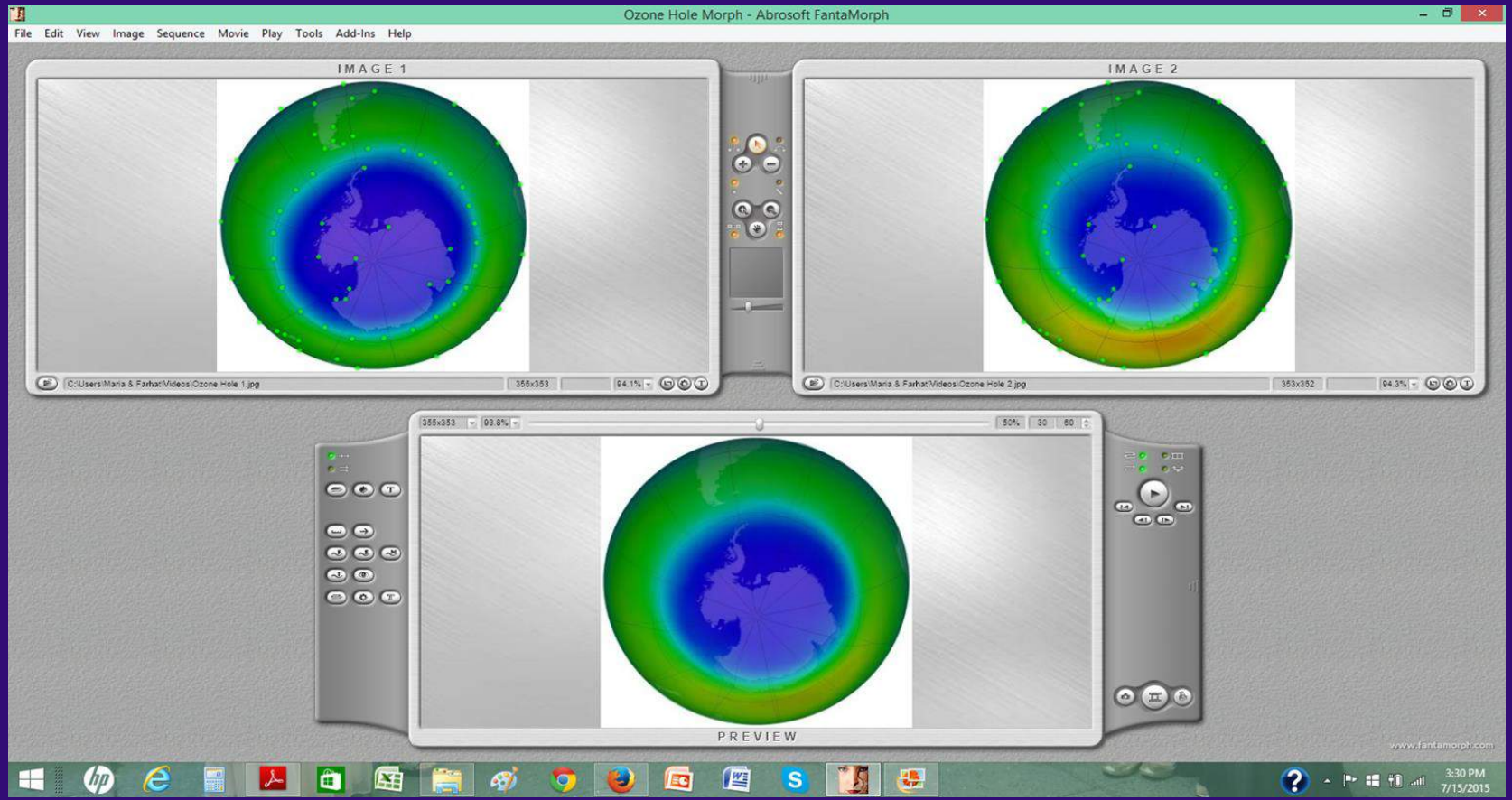
Procedure to Create a Morph

1. Input points on defining features in image 1.
   - Points = vertices.
2. Create a mesh of triangles (triangulation) from the points.
   - Both images 1 and 2 have a congruent mesh of triangles.
3. Move the points in the second image to desirable/corresponding locations.
   - Avoid overlapping triangles.

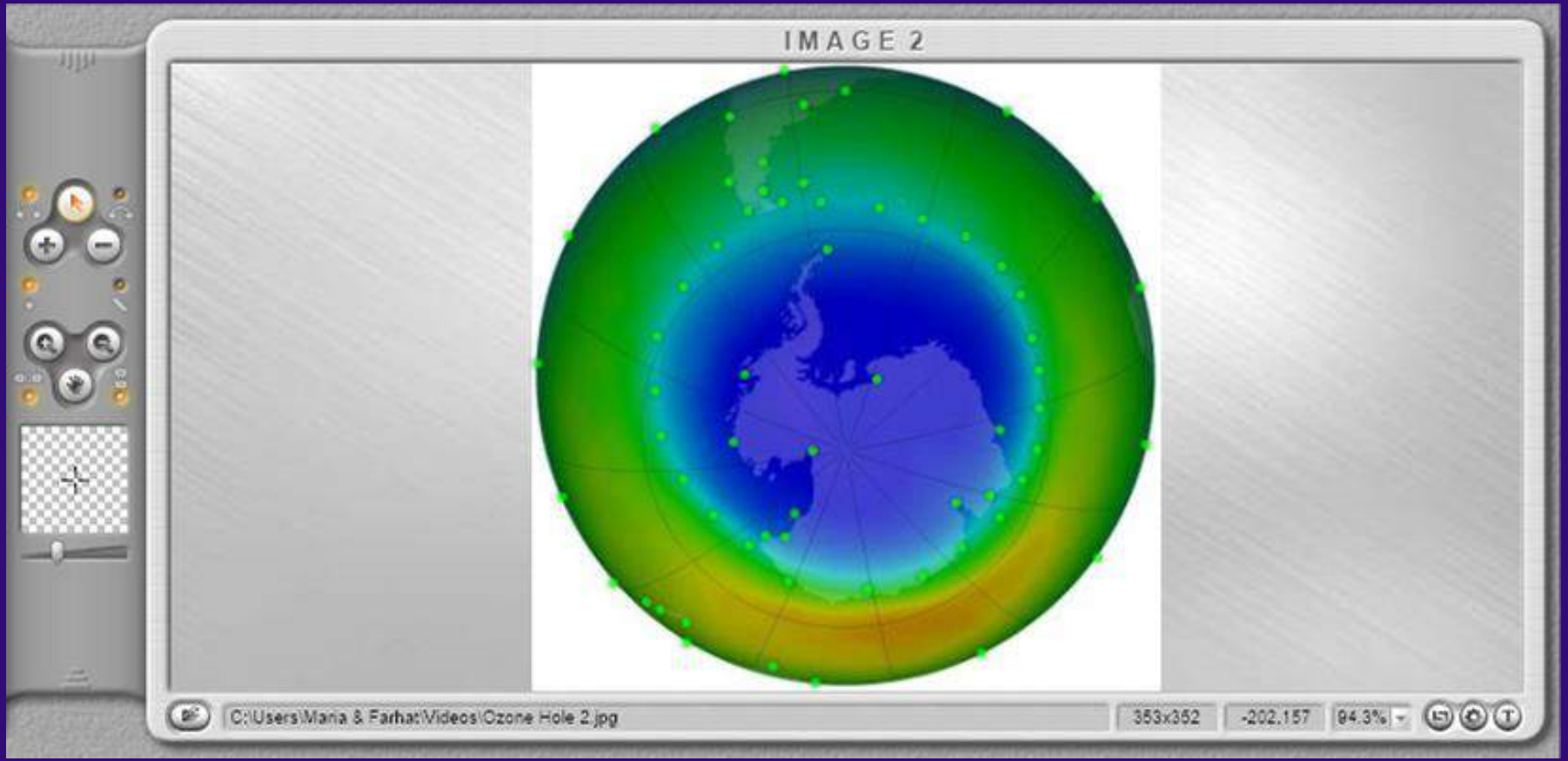
**When creating a morph, choose images in which the objects are positioned similarly.

The Result……

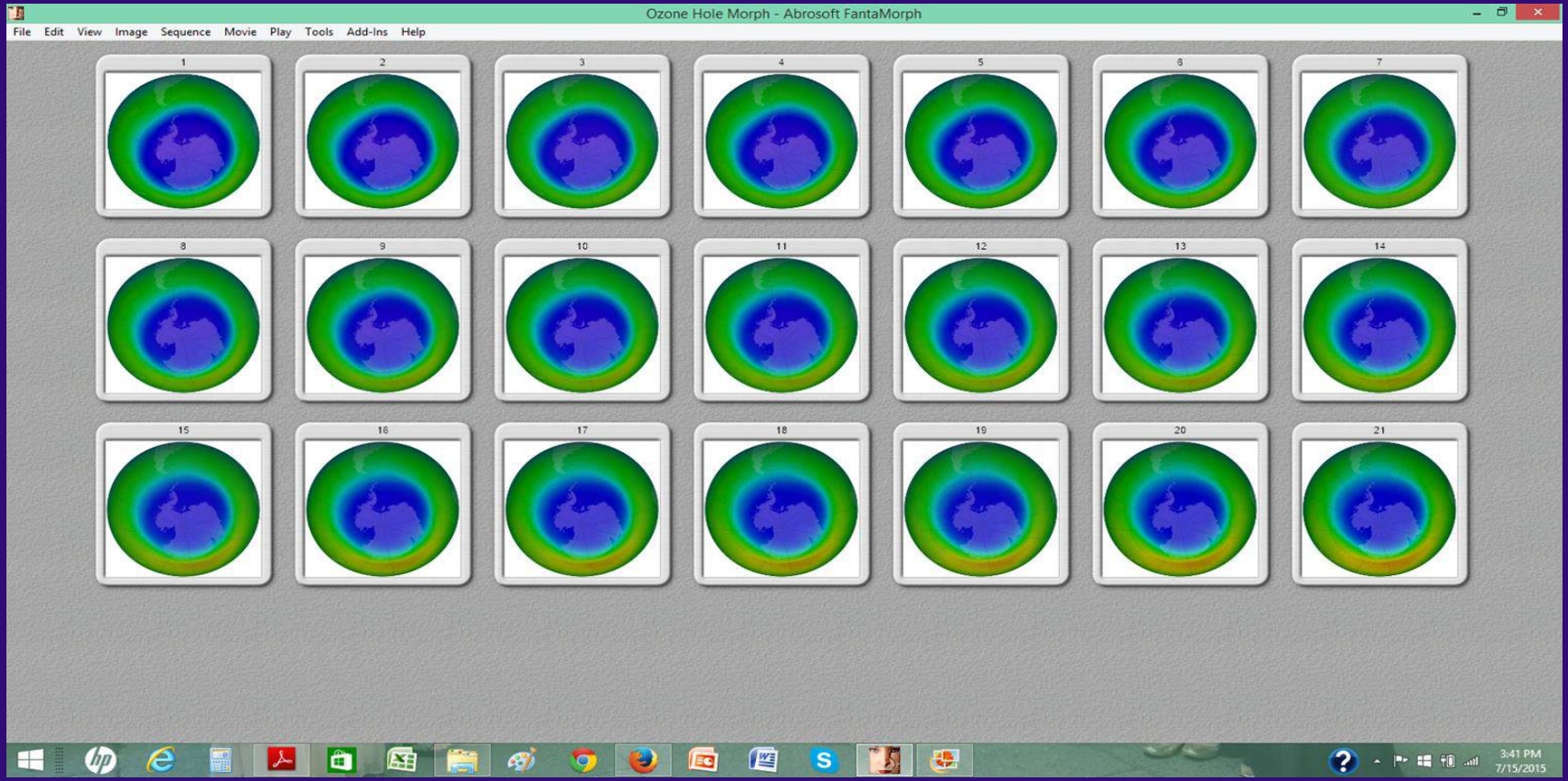
Image Morphing/Warping Software: *FantaMorph*



Morph/Warp Window:



Frame Sequence:

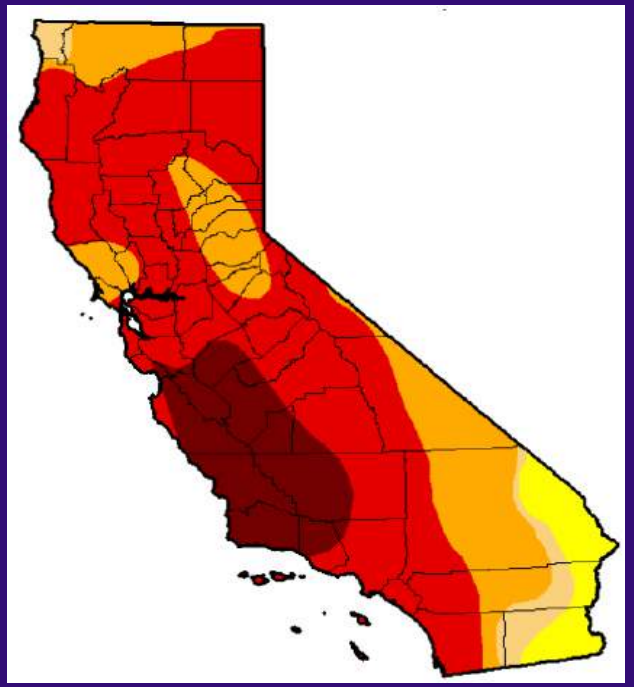


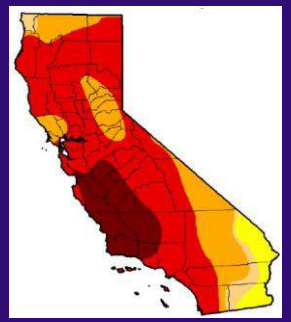
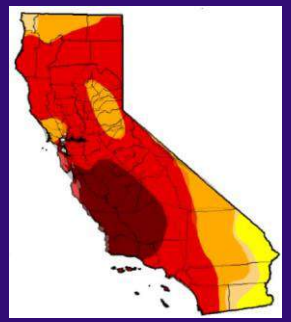
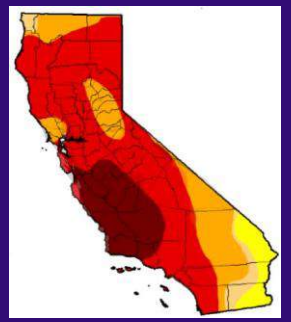
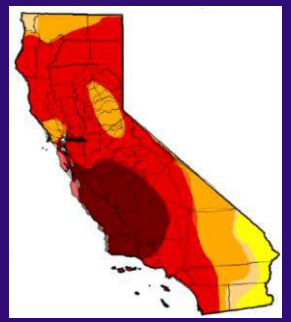
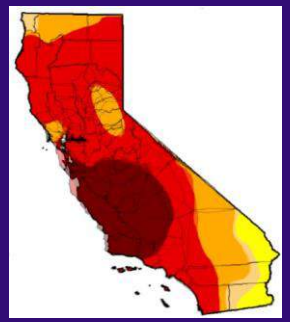
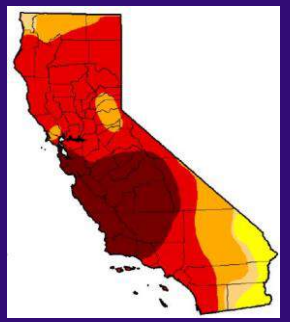
The team successfully created a series of warps and morphs.

Here are some examples…..

California Drought Morph

- **M**apping the course of natural disasters, such as droughts.
- Drought intensity in California over a one week span.
- Dark red/maroon = highest drought intensity.



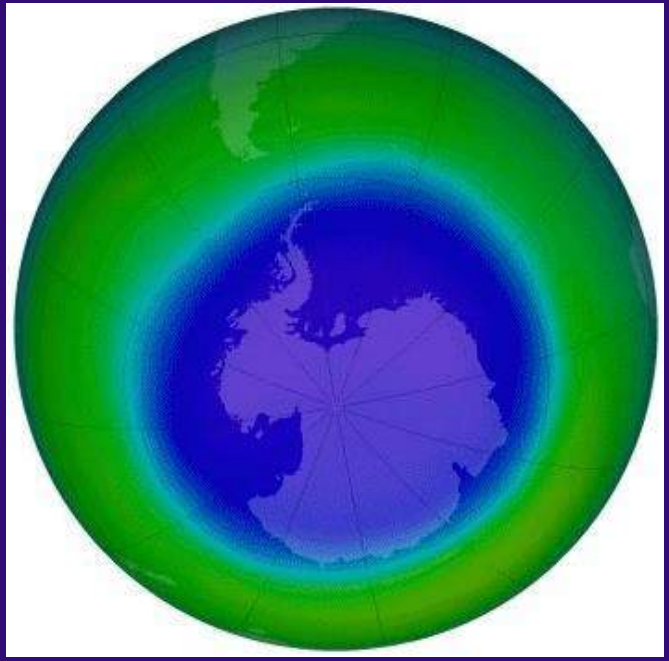







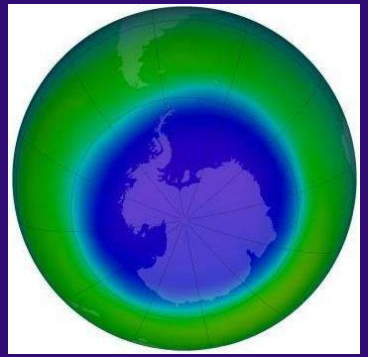
Super Nova Morph

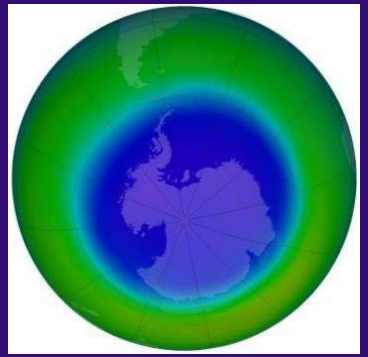
- February 23, 1987.
- A star exploded in the Large Magellanic Cloud - a nearby galaxy.
- Though the morph created here assumes a constant rate of explosion, other non-linear morphs can remedy this and provide more accurate representations.

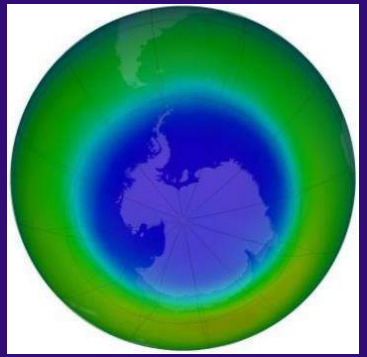
Ozone Hole Morph

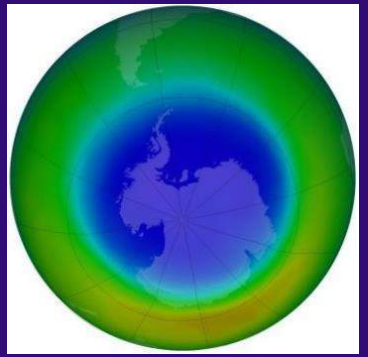
- Some changes in a vector are hard to view with the naked eye.
- Morphing images that depict slight changes can create easier-to-view depictions of those vectors.
- Slight decrease in the size of the ozone hole from September 2006 to September 2012.

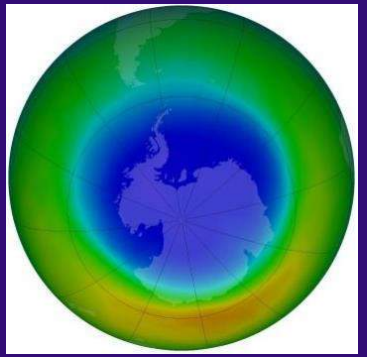










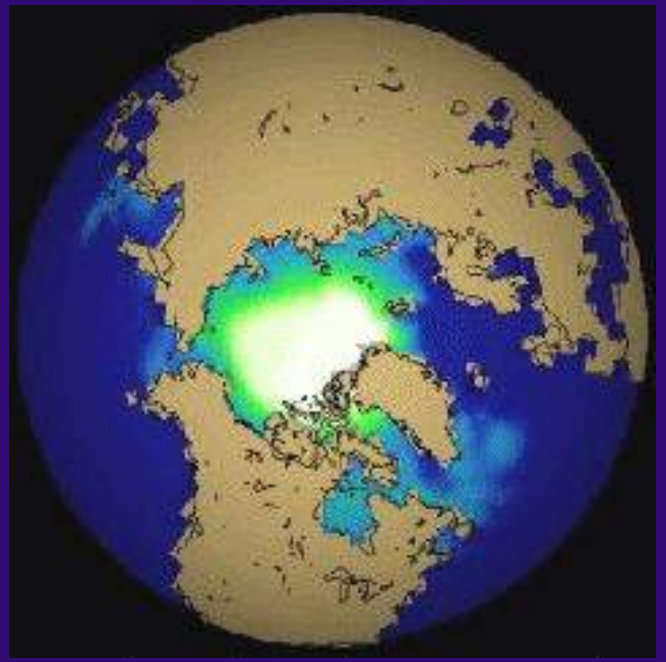


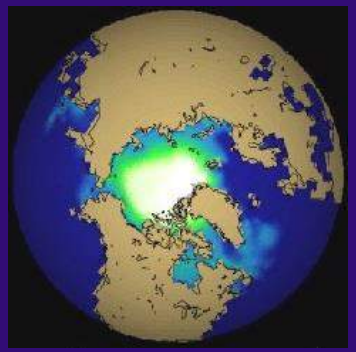
Morphing People

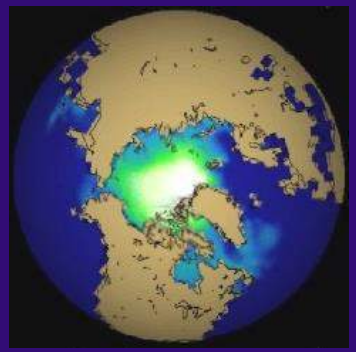
- Can be used to depict the gradual aging process of one human.
- Morphing family members or any humans:
  - Can be used to compare similarities.
  - Can be used to compare differences in facial structure and body form.

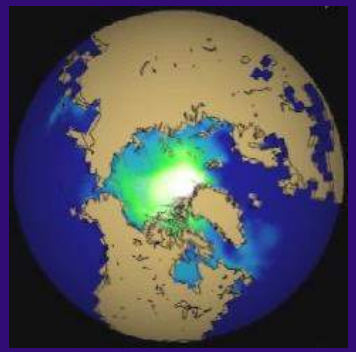
Arctic Sea Ice Morph

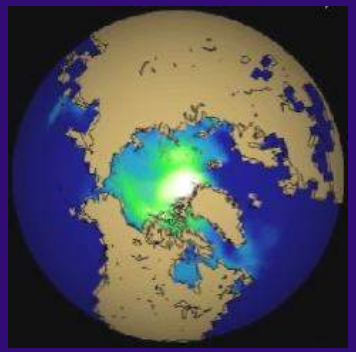
- Comparison depicting the gradual changes in sea ice thickness.
- Dramatic decrease of sea ice thickness in the Arctic region from the 1950s to its projected thickness in 2050.
- Morphing techniques can depict environmental changes.

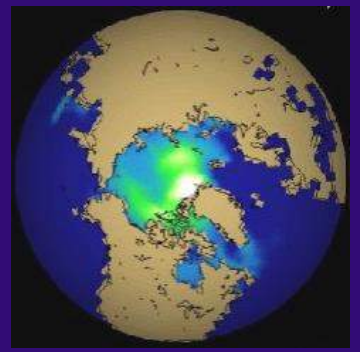






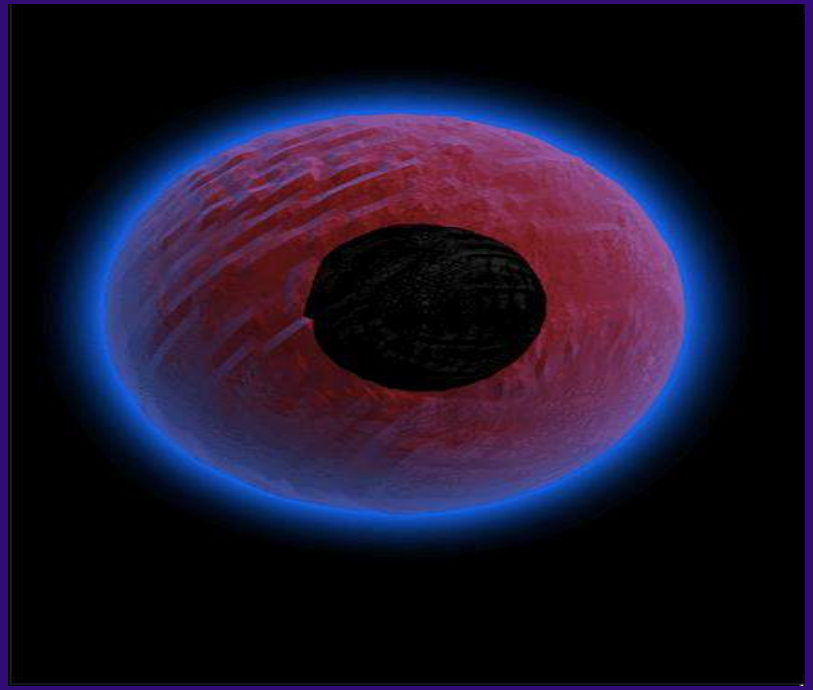


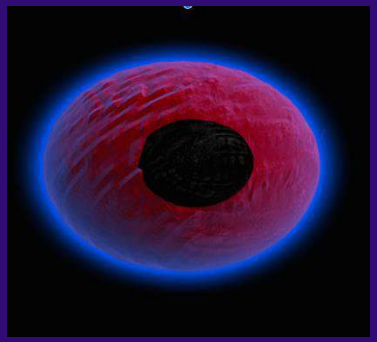


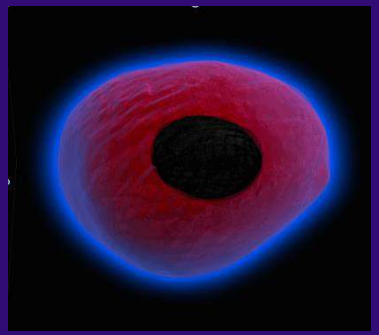


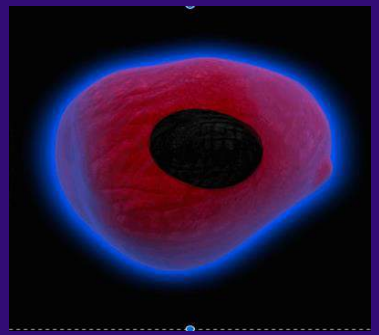
Cancer Cell Morph

- The transformation of a healthy cell into a cancer cell.
- Morphs can be used to show change in biological processes.
- Cancer is a disease that is caused by the uncontrollable cell division of abnormal cells.



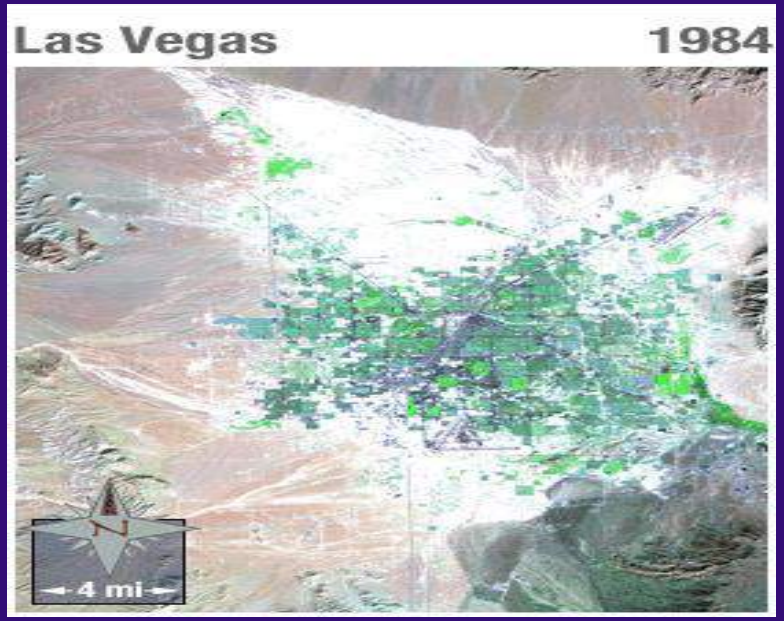


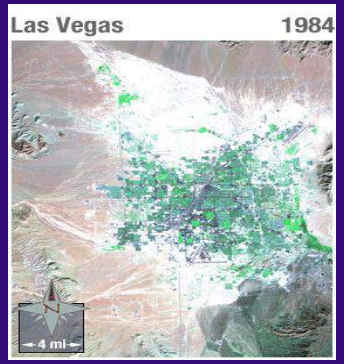


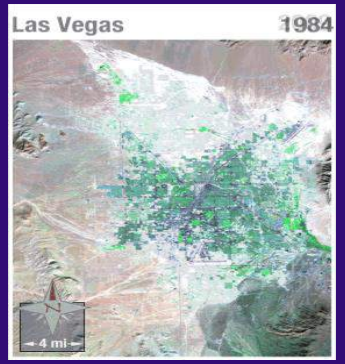


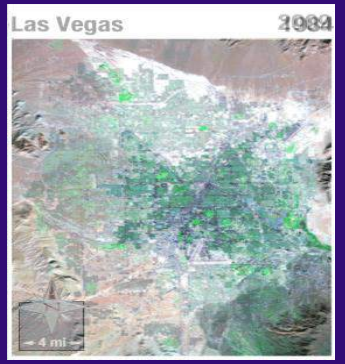
Urban Growth Morph

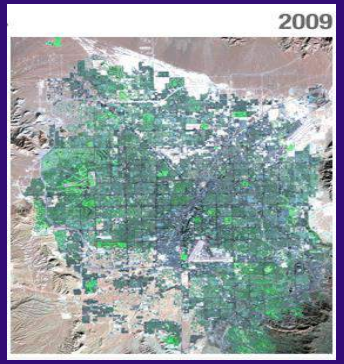
We can use warp and morphing as a modeling tool to predict urban growth vectors.







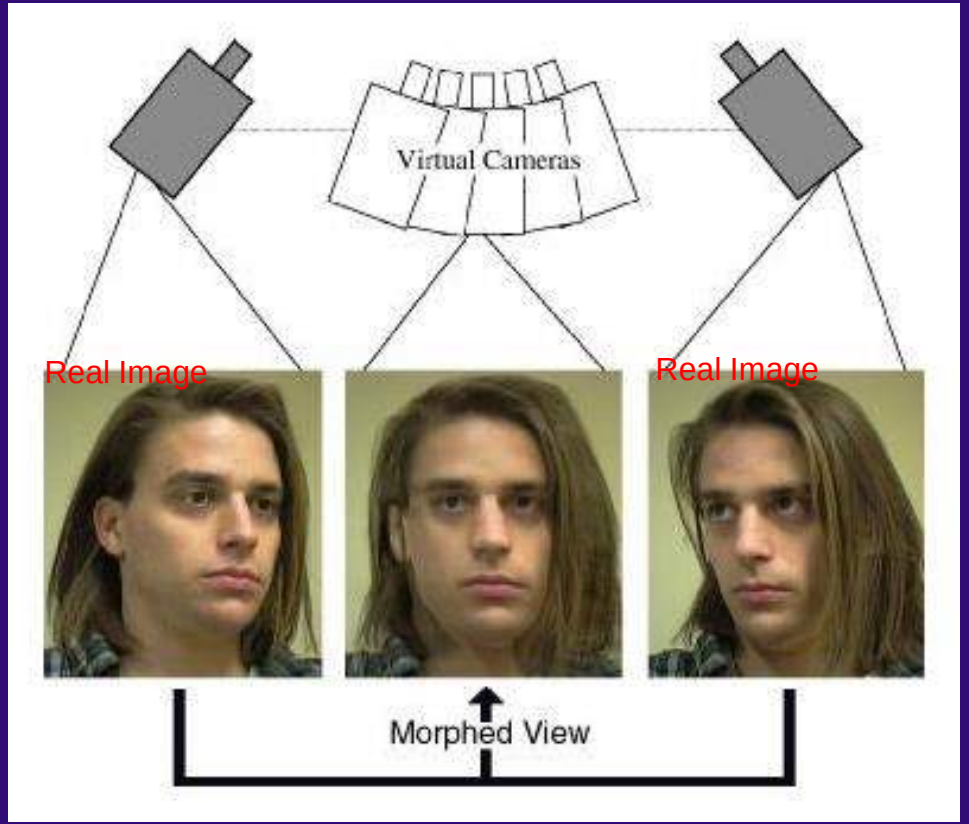




Creating a 3D illusion from 2D images

- Reminiscent of utilizing a virtual camera.
- After taking images from various angles of an object, one can separately morph consecutive images.
- Collate these individual morphs.

***As a result, when observing the result - the “master” morph - one is given the illusion of viewing a virtual camera.



This significant technique can be used in an array of fields; we can create virtual camera and 3D depictions of landforms, planets, humans, animals, objects, locations, inventions, machines, and more.

3D illusion created from 1 image:

Creating a 3D illusion from 2 images:

Relevance to NASA

- Depict environmental changes on Earth.
- Transforming 2D satellite images into 3D virtual camera views.
- Combine images captured by different satellites to form more uniform and cohesive pictures of objects.
- Depict the gradual process that an object undergoes within the span of time the two images are taken.
- Emphasize any slight or easily overlooked changes an object experiences.

Limitations

- Some morphs can only be instigated in a linear fashion.
- The change between each frame in the morph happens at a constant rate.
- May not represent realistic or natural processes.
- If an object undergoes dramatic changes in which it loses or gain new parts, it is inevitable that the triangles will overlap at some point of the morphing process. As a result, it is simply impossible for some objects to be morphed between images.

Future Work

- Image mosaics
- Correction of image distortion
- Studying warps/morphs that happen in a trigonometric or quadratic process
- Creating projections
- Morphing/warping different parts of images at different rates

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Thank You!

Any Questions?